The University of the West Indies

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PROFESSIONAL DEVELOPMENT PROGRAMME: COASTAL INFRASTRUCTURE DESIGN, CONSTRUCTION AND MAINTENANCE

A COURSE IN

COASTAL DEFENSE SYSTEMS I

CHAPTER 5

COASTAL PROCESSES: WAVES

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5. Coastal Processes: WAVES

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5.1 Introduction.

Wave action is obviously a major factor in coastal engineering design. Much is known about wave mechanics when the wave **height** and **period** (or **length**) are known. In shallow water the properties of waves change; they change height and their direction of travel, which must be included in design calculations. However, waves generated by winds blowing over the ocean surface are not of the same height and period, they are **random waves** for which probability/statistical models have to be used.

This section of the notes discusses these three topics: linear waves, random waves and waves in shallow water.

5.2 Linear (Uniform) Wave Theory.

Figure 1 gives the general definitions for two-dimensional, linear water wave theory for which the following notation is needed:

x,y are Cartesian co-ordinates with y = 0 at the still water level (positive upwards)

$\eta(x,t)$ = the free water surface; t = time
u,v, = velocity components in the x,y directions, respectively
$\phi(x,y,t)$ = the two-dimensional velocity potential
ρ = the fluid density; g = gravitational acceleration
a = wave amplitude = H/2; H = wave height
k = wave number = $2\pi/L$; L = wave length
σ = wave frequency = $2\pi/T$; T = wave period
d = mean water depth; C = wave celerity = L/T

Linear wave theory is a solution of the Laplace equation:

$$\delta^2\phi/\delta x^2 + \delta^2\phi/\delta y^2 = 0 \qquad 5.1$$

The particular flow in any condition is determined by the boundary conditions, in this case specific boundary conditions at the free surface of the fluid and at the bottom.
(The details can be found in any good text on wave theories)

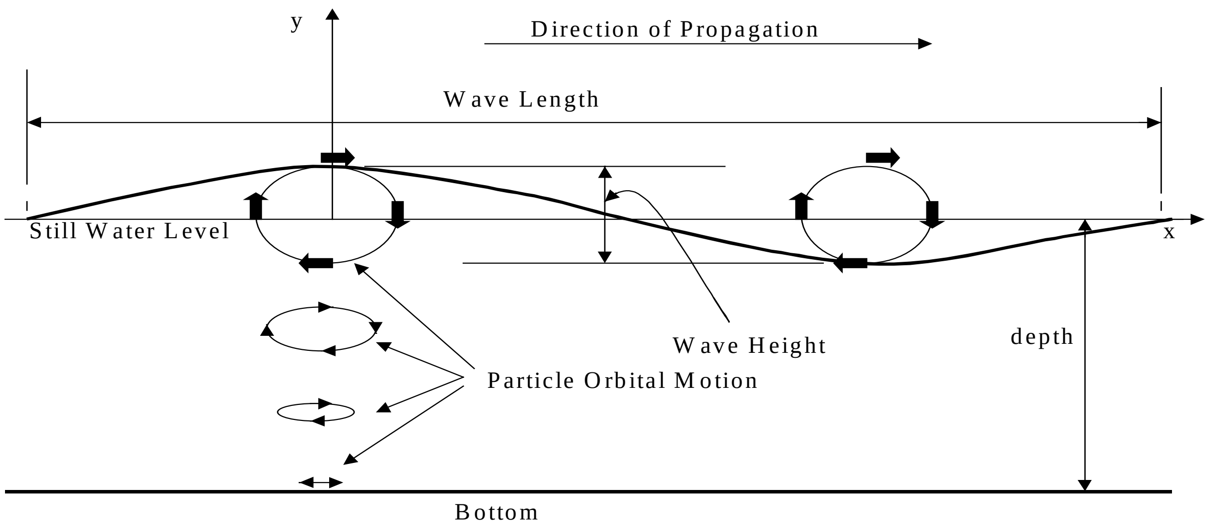

Figure 1. Definitions for Linear Water Wave Theory

The solution for the potential function satisfying the Laplace equation (1) subject to the boundary conditions is:

$$\phi = (ga/\sigma).\ \frac{\cosh k(y + d)}{\cosh\ kd}.\ \sin (kx - \sigma t) \qquad 5.2$$

$$= (gHT/4\pi).\ \frac{\cosh [(2\pi/L)(y + d)]}{\cosh (2\pi d/L)}.\ \sin 2\pi(x/L - t/T) \qquad 5.3$$

$$\eta = a \cos(kx - \sigma t) = (H/2) \cos 2\pi(x/L - t/T) \qquad 5.4$$

$$\sigma = (gk \tanh kd)^{1/2} \qquad 5.5$$

or $$L = (gT^2/2\pi) \tanh (2\pi d/L) \qquad 5.6$$

or $$C = ((gL/2\pi) \tanh (2\pi d/L))^{1/2} \qquad 5.7$$

In "deep" water, $d/L > 0.5$, $\tanh (2\pi d/L) \approx 1.0$

$$\therefore \; L_o = gT^2/2\pi = 1.56T^2 \qquad 5.8$$

where subscript "o" denotes deep water.

In "shallow" water, $d/L < 0.04$, $\tanh(2\pi d/L) \approx 2\pi d/L$

$$\therefore \; L = T(gd)^{1/2}; \; C = \sqrt{gd} \qquad 5.9$$

For all depths the wave length, L can be found by iteration from:

$$d/L_O = d/L \tanh(2\pi d/L) \qquad 5.10$$

Particle Velocities.

From the derivation of the Laplace equation, for irrotational flow,

$$u = \delta\phi/\delta x \quad \text{and} \quad v = \delta\phi/\delta y \qquad 5.11$$

so that from equation 2 the horizontal and vertical velocities of flow are given by:

$$u = (\pi H/T).\ \frac{\cosh[2\pi(y+d)/L]}{\sinh 2\pi d/L}.\ \cos 2\pi(x/L - t/T) \qquad 5.12$$

$$v = (\pi H/T).\ \frac{\sinh[2\pi(y+d)/L]}{\sinh 2\pi d/L}.\ \sin 2\pi(x/L - t/T) \qquad 5.13$$

In "deep" water these simplify to:

$$u = (\pi H/T).\ \exp(2\pi y/L_O).\ \cos 2\pi(x/L - t/T) \qquad 5.14$$

$$v = (\pi H/T).\ \exp(2\pi y/L_O).\ \sin 2\pi(x/L - t/T) \qquad 5.15$$

Note that y is measured positive upwards from the still water level.

Pressure.

In wave motion the pressure distribution in the vertical is no longer hydrostatic and is given by:

$$p/\rho g = \frac{\cosh 2\pi[(y+d)/L]}{\cosh 2\pi d/L}\ .\ \eta - y \qquad 5.16$$

The $\cosh 2\pi[(y + d)/L] / \cosh 2\pi d/L$ term is known as the "pressure response factor" which tends to zero as y increases negatively, important when using pressure transducers for wave recording.

Energy.

The total energy per wave per unit width of crest, E, is:

$$E = \rho g H^2 L/8$$

5.17

Note that wave energy is proportional to the SQUARE of the wave height.

Group Velocity.

Group velocity, C_G, is defined as the velocity at which wave energy is transmitted. Physically this can be seen if a group of, say, five waves is generated in a laboratory channel. The leading wave will disappear but a new wave will be created at the rear of the group, so there will always be five waves. Thus the group travels at a slower speed than the individual waves within it.

$$C_G = nC, \quad n = ½ [1 + (4\pi d/L)/(\sinh 4\pi d/L)] \qquad 5.18$$

Power.

The mean power transmitted per unit width of crest, P, is given by:

$$P = C_G \rho g H^2/8 \qquad 5.19$$

Non-Linear Wave Theories.

As noted above the boundary conditions used to obtain a solution for wave motion were linearised, that is, applied at y = 0 not on the free water surface, y = η, hence the term Linear (or Airy) Wave Theory. For a non-linear solution the free surface boundary conditions have to be applied at that free surface, η. But η is unknown! Therefore solutions have been developed, notably by **Stokes**, in series form for which the coefficients of the series can be derived.
Thus, the free-surface is given by:

$$\eta = a \cos(\theta) + b \cos(2\theta) + c \cos(3\theta) + d \cos(4\theta) + e \cos(5\theta) \ldots\ldots 5.20$$

To obtain a solution to, say, third order, terms greater than order three are ignored. Taken to first order the solution is, of course, a linear wave. Another wave theory applicable in shallow water is **Cnoidal Wave Theory**. Solutions are given in terms of elliptic integrals of the first kind; the solution at one limit is identical with linear wave theory and at the other is identical to **Solitary Wave Theory.** As the name implies, the latter is a single wave with no trough and the mass of water moving entirely in the x direction.

More recently, numerical solutions for wave motion have been established and in offshore engineering it is common to see numerical solutions up to 18^{th} or 25^{th} order being used to obtain velocity and acceleration information for the derivation of wave loads on offshore structures.
Figure 2, based on the U.S. Army corps of Engineers “Shore Protection Manual” 1984, indicates the preferred theory for given wave parameters, but note that the figure is illustrative only.

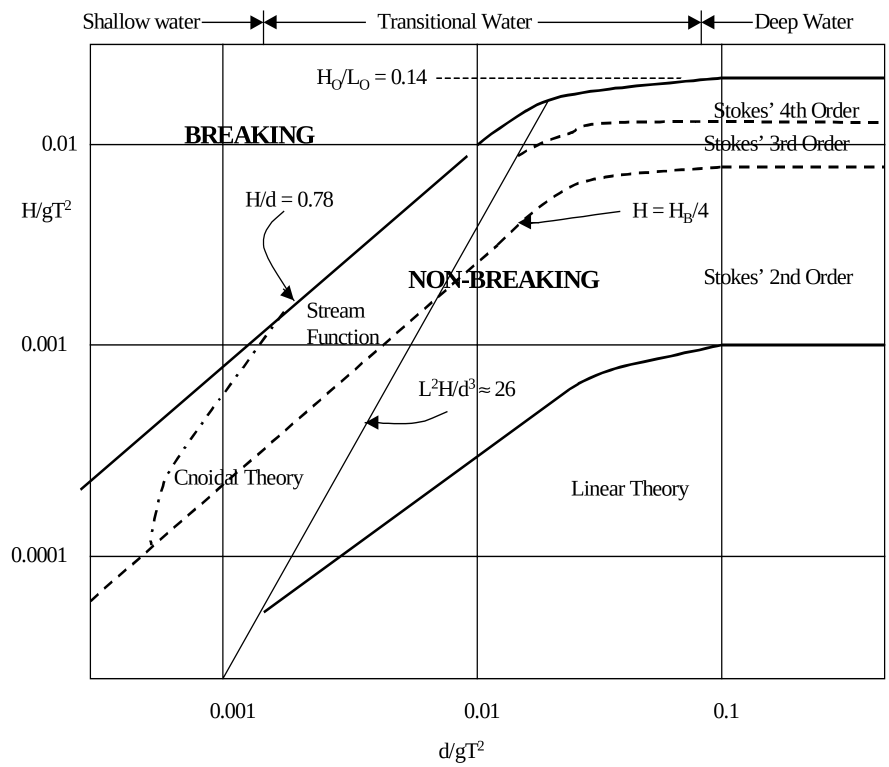

Figure 2. Parameter Space for Wave Theories - illustrative only.

5.2. RANDOM WAVES

5.2.1. Introduction.

Waves generated by winds blowing over the ocean have a range of wave periods,
T (s), or wave frequency, ω (radians/s) and are clearly not of the same height, H

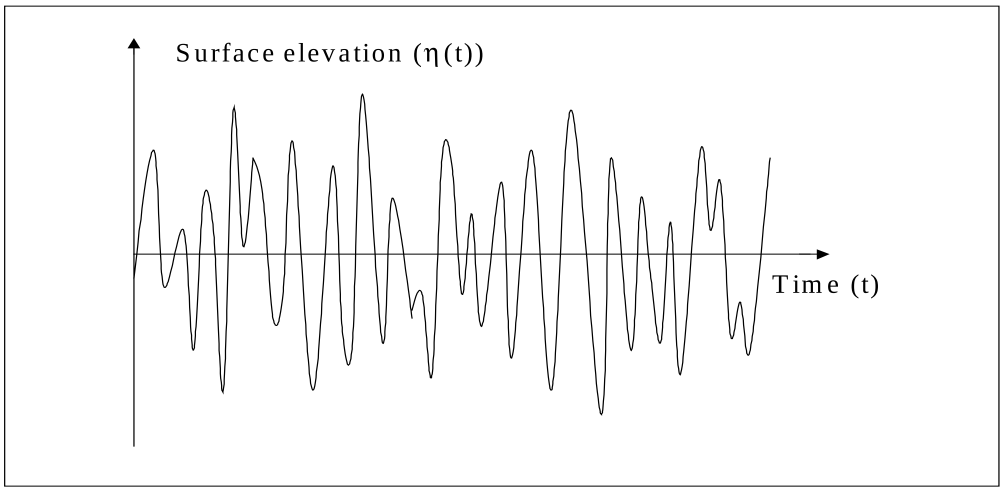

Figure 1 A Typical Wave Record

(m). These variations arise initially from the random nature of pressure pulses acting on the water surface and the subsequent, complex growth of the perturbations in that free surface.

Based on linear wave theory it is clear that waves with longer periods travel at a higher celerity that those with a shorter period. Thus, as the waves travel away from the area in which they are generated the longer period waves travel faster and the water surface elevation is more periodic, usually being termed "swell". Within or close to the generation area the sea surface elevation is a complex three-dimensional process with short-crested waves travelling in a range of directions relative to the wind direction.

A record of such waves will look like Figure 1.

The surface elevation is measured relative to still water level and a second record taken at some later time would have a similar appearance, i.e., its general properties would not change - it is a stationary process. Any linear derivatives of the surface elevation, for example, the water particle velocity or acceleration would have similar appearances.

For design purposes it is necessary to quantify this in some way. This can be done in two ways: in terms of the probability properties of the record or in terms of its frequency content, specifically the distribution of energy as a function of frequency, called the wave spectrum.

5.2.1 Wind Generation of Waves and Wave Prediction.

The physical processes in the generation of wave by wind are complex. In brief,
the turbulent fluctuations in the wind result in pressure fluctuations being imposed on the water surface which deforms in the form of ripples. If these ripples travel at the same speed as the pressure fluctuation there will be a continuous transfer of energy from wind to water. This process often results in ripples travelling at an angle to the mean wind direction giving a diamond shaped pattern to the water surface.

As the ripples grow they begin to deform the air flow above them; this causes a change in the pressure distribution on the water surface and the rate of wave growth increases rapidly. Complex circulation patterns are set up in the air flow and the wave height continues to increase. As the height and, particularly, the length of the waves increases the speed of the wave also increases. Eventually the wave travels at the same speed as the wind and thereafter there is no transfer of energy from wind to water, at least, in a linear wave theory model. The Sea State is said to be **"fully developed".** Based on non-linear wave theories it is possible for energy to be transferred between waves which allows longer, larger waves to grow further by the addition of energy from the shorter, slower waves.

Obviously the heights and periods of the waves depend on the speed of the wind, the distance over which the wind acts on the waves, termed the **"fetch",** and the **duration** of the wind. Forecasting wave conditions from wind data is a complex process, especially if the wind speed and direction change with time - which is a common event. Numerical models exist for such wave prediction but for design purposes it is possible to use a simpler model, noting that generally one needs to predict waves in extreme, design conditions.

The three input parameters needed for wave prediction are:

the **WIND SPEED** (m/s) or knots, U, (at 10m above the water or ground surface)

the **FETCH** (m) or nautical miles, F, (one Nautical mile = 1852m) and

the **DURATION**, D (hours).

In these notes the U.S.Army Corps of Engineers, Shore Protection Manual (SPM) model is used to illustrate the prediction process.

From SPM the following relationships can be used to predict wave conditions:

Where the wave conditions are **FETCH LIMITED**:

$$H_S = 1.616 \times 10^{-2} U_A F^{1/2} \qquad 5.21$$

$$T_M = 6.238 \times 10^{-1} (U_A F)^{1/3} \qquad 5.22$$

$$t = 8.93 \times 10^{-1} (F^2 / U_A)^{1/3} \qquad 5.23$$

In **FULLY DEVELOPED** conditions:

$$H_S = 2.4821 \times 10^{-2} U_A^2; \quad T_M = 0.83 U_A; \quad t = 2.027 U_A \qquad 5.24$$

where H_S is in m, T_M is in seconds, U_A is in m/s, F is in km., and t, the duration required to reach fully developed or fetch limited conditions, is in hours. U_A is an "adjusted wind speed"

$$U_A = 0.71 U^{1.23} \qquad 5.25$$

where U is the actual wind speed in m/s.

Note that 1 Nautical Mile = 1.852km.
Functional relationships for the influence of limited durations are less well known but this case does not often arise in design.
In shallow waters the energy transfer from the wind to the waves is limited by bed friction. Algebraic relationships are available for these conditions but the most convenient way to forecast wave conditions is to use the graphs in SPM - for both deep water and shallow water cases, page 3-39 *et seq*. in the 1984 edition.

5.2.3. Probablilistic Properties of Random Waves.

Observation of waves shows that they are rarely uniform - except in the case of "swell" waves - see later. Therefore two parallel and inter-linked models of random waves have been developed, one based on probability theory and the other using time series analysis.

Figure 4 illustrates part of a wave record of total duration T. Taking a small interval, $d\eta$, in the surface elevation, the record can be analysed to determine the total time for which

$$(\eta_j - d\eta/2) < \eta \leq (\eta_j + d\eta/2) \qquad 5.26$$

i.e., $$T(\eta_j) = \Sigma\, t_i \quad \text{for } i = 1 \text{ to } n \qquad 5.27$$

The pdf of η can then be estimated as: $p(\eta_j)d\eta = \lim\ T(\eta_j)/T \quad \text{as } T \Rightarrow \infty$

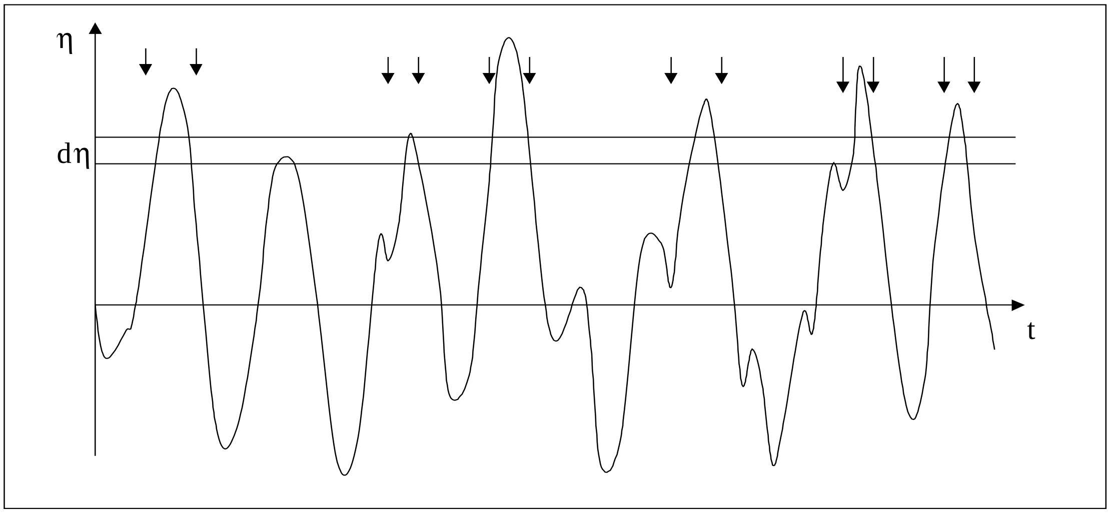

Figure 4. Sampling from a Wave Record.

Based on the Central Limit Theorem - which states, in effect, that the value of a variable resulting from a large number of independent causes has **a Gaussian or Normal pdf** - and verified by full-scale measurements then:

$$p(\eta) = (1/(\sqrt{2\pi} \, . \, \sigma_\eta)). \exp[-\eta^2 / 2\sigma_\eta^2] \qquad 5.28$$

here, $\bar{\eta}$, the mean water level, is taken as zero, and σ_η is the **STANDARD DEVIATION**.

The probability distribution is found from tables of the Normal Integral, e.g.,

η	$P(\eta)$
0	0.5
σ_η	0.8413
$2\sigma_\eta$	0.9772
$3\sigma_\eta$	0.9987

We now consider wave heights, H, usually defined as the vertical distance between the highest crest elevation and the lowest trough elevation between two successive
up-crossings of the mean water level, i.e., up-crossings of zero.

Wave Heights.
Based on joint probability density functions of η and its time derivatives - to specify crests and troughs - it is possible to derive the probability density and distribution functions for H in the form:

$$P(H) = 1 - \exp[-H^2/8\sigma_\eta^2] \qquad 5.29$$

$$p(H) = H/4\sigma_\eta^2 \exp[-H^2/8\sigma_\eta^2] \qquad 5.30$$

This pdf is known as the **RAYLEIGH probability density function** from which various useful properties of H can be derived:

Mean Wave Height, $\overline{H} = 2.51\sigma_\eta$ 5.31

Root-mean-square Wave Height, $H_{rms} = 2.83\sigma_\eta$ 5.32

Average height of the highest 1/3rd, $H_{1/3} = 4.00\sigma_\eta$ 5.33

Average height of the highest 1/10th, $H_{1/10} = 5.08\sigma_\eta$ 5.34

Average height of the highest 1/100th, $H_{1/100} = 6.67\sigma_\eta$ 5.35

$H_{1/3}$ is often referred to as the **Significant Wave Height, H_S,** and is close to the "height" of random waves that would be reported by an observer. As is shown later in these notes, H_S is very frequently used as a parameter to represent the wave conditions for design calculations.
An additional useful relationship is that which given the "expected value of the highest wave in N waves":

$$E[H_m] = 0.5H_S (2 \ln N)^{1/2} \qquad 5.36$$

The most common parameter for the wave period in a random sea is T_Z, the zero-crossing period, defined as the average period between successive up-crossing of the zero or mean water level. In terms of parameters of the time history of water surface elevation it can be estimated by:

$$E[T_Z] = 2\pi (m'_0/m'_2)^{1/2} \qquad 5.37$$

Where m'_0 and m'_2 are the zeroth and second moments of the spectrum - see the following section of these notes.
Note that the T_M - the period of the spectral peak (see below) - derived from wave forecasting curves equals 1.05 T_Z although the probabilistic properties of wave periods are not well understood. T_Z, the zero-crossing period, is most frequently

used in design but one would always check the sensitivity of design calculations to variations in wave period.
Given a convenient way of parameterising wave heights based on the Rayleigh pdf of H, it is standard practise to characterise wave conditions in terms of H_S and $T_{Z.}$ Normally,
wave records for a particular site are based on ten or fifteen minute samples recorded every three hours for at least a year. This gives eight records per day, 2929 records per year, provided there are no instrument failures or thefts! Thus the highest significant wave height in a year has a probability of 1/2920. However, for design purposes it is necessary to estimate the maximum significant wave height in 50 or 100 years, which requires the application of extreme value statistics.

Design Wave Prediction.
All designs require the specification of extreme conditions. In coastal engineering it is usual to design for the worst wave conditions expected to be equalled or exceed on average once in N years, where N is often 50 or, in offshore engineering, 100.
Available wave data will typically be one year's records, i.e., a total of 2920 values of H_S,T_Z pairs although with modern wave recorders a dominant wave direction may also be available for each record.
The following table is based on one year's wave records (incomplete) from a weather station in the North Sea.

H_S(m)	T_Z(s)	Number of Observations.
0.0- 0.6	5.75	96
0.6 -1.2	5.76	402
1.2 -1.8	6.33	389
1.8 - 2.4	6.72	321
2.4 - 3.0	7.05	245
3.0 - 3.6	7.24	161
3.6 - 4.2	7.62	132
4.2 - 4.8	8.17	70
4.8 - 5.4	8.17	46
5.4 - 6.0	8.39	23
6.0 - 6.6	9.51	12
6.6 - 7.2	10.00	11
7.2 - 7.8	10.10	8
7.8 - 8.4	8.50	2
8.4 - 9.0	10.00	4
9.0 - 9.6	13.50	2
> 9.6		0
	Total	1924

This is a convenient way of presenting the data but, of course, it loses the sequential or time history properties of the data which may be important if further statistics such as the average time between given wave height thresholds is required - for construction planning and operations, for example.

Although we have a bi-variate data set, (H_S,T_Z), there is no theoretical model for a bi-variate probability density function, (pdf). Engineering design usually concentrates on wave height and therefore the marginal pdf of H_S, i.e., the pdf of H_S ignoring T_Z is most important.
Two commonly used probability distribution functions for extreme values are used in the prediction of extreme waves, namely the Gumbell and Weibull distributions.

The Gumbell Extreme Value Distribution, (Extreme Value Distribution Type 1).

$$P(x) = \exp\{ -\exp[-\alpha(x -u)]\} \qquad 5.38$$

EV1 is clearly a TWO parameter distribution; knowing α and u the function can be plotted and these two parameters are related to the mean, $\bar{x}$, and the standard deviation,σ, by:

$$\alpha = \pi/(2.45\sigma); \quad u = \bar{x} - 0.5772/\alpha \qquad 5.39$$

Taking logarithms twice:

$$\alpha(x - u) = -\ln[-\ln P(x)] \qquad 5.40$$

so that a plot of x versus -ln[-lnP(x)] results in a straight line.

The Weibull Extreme Value Distribution, (Extreme Value Distribution Type 3).

$$P(x) = 1 - \exp[- ((x-A)/B)^c)] \qquad 5.41$$

This is a THREE parameter distribution, A, B and C, which are related to the mean and standard deviation by:

$$\bar{x} = A + B\,\Gamma(1 + 1/C); \quad \sigma = B\,[\Gamma(1 + 2/C) - \Gamma^2(1 + 1/C)]^{1/2} \qquad 2.42$$

where Γ denotes the Gamma function. (obtainable from tables. $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx$)

Taking logarithms twice:

$$\ln.(\ln[(1 - P(x))^{-1} = C.\ln (x - A) - C.\ln B \qquad 5.43$$

and a plot of the L.H.S. versus the R.H.S. would be a straight line. For wave data, parameter A is usually very small so an initial plot is performed assuming that A = 0. The goodness of the fit can then be used to estimate a value of A and the data re-plotted.
There are several ways of "fitting" data to a presumed line, e.g., methods of moments, maximum likelihood, and least-squares - the latter being the equivalent of fitting "by eye". Therefore if $\bar{x}$, (i.e. $\bar{H}_S$) and σ (i.e. σ_{HS}) are calculated from the data the parameters of the distributions can be found. The data, summated sequentially to give $P(H_S)$, can then be plotted on the same graph to judge the goodness of fit.

If the wave data is of limited duration, one or two years, it is necessary to check, using a longer sample of wind data, that the conditions during the wave recording were "average". Note also that there is nothing in these models which accounts for shallow water depths and, of course, in shallow water the maximum wave height will be limited by breaking - thus giving an upper limit to long-term wave heights.

Time Series Analysis and Wave Spectra.

Although the probabilistic model is the basis of determining design wave conditions it uses general parameters, H_S and T_Z, to describe the sea state. Often in design, especially for systems which respond dynamically, more information on the waves heights associated with different wave periods is needed. It is here that time series analysis is applied.

A single sinusoidal wave, travelling in the x-direction, can be defined in terms of its period and height:

$$\eta(x,t) = a \sin(kx - \omega t + \theta) = (H/2) \sin 2\pi(x/L - t/T + \theta) \quad 5.44$$

where a = the wave amplitude = H/2 (m)
k = the wave number = $2\pi/L$ (radians/m)
ω = the wave frequency = $2\pi/T$ (radians/s)
[f is also used for wave frequency = 1/T (Hz)]
and θ is a phase angle.

Noting that the energy per unit surface area for a linear wave is $\rho g a^2/2$, the wave can be defined by its height, H, and its period, T, or by a^2, proportional to its energy, and ω, its frequency.
On a graph of wave energy against frequency this single wave would be represented by one point, as shown by the arrow in figure 3.
A two-dimensional, random sea, (all waves travelling in the x-direction) can be considered as the summation of many individual linear waves:

$$\eta(x,t) = \sum_{n=1}^{n=\infty} a_n \sin(k_n x - \omega_n t + \theta_n) \quad 5.45$$

It is the assumption that the θ_n are independent and uniformly distributed between zero and 2π which results in the Gaussian pdf. property for water surface elevation, based on the Central Limit Theorem.

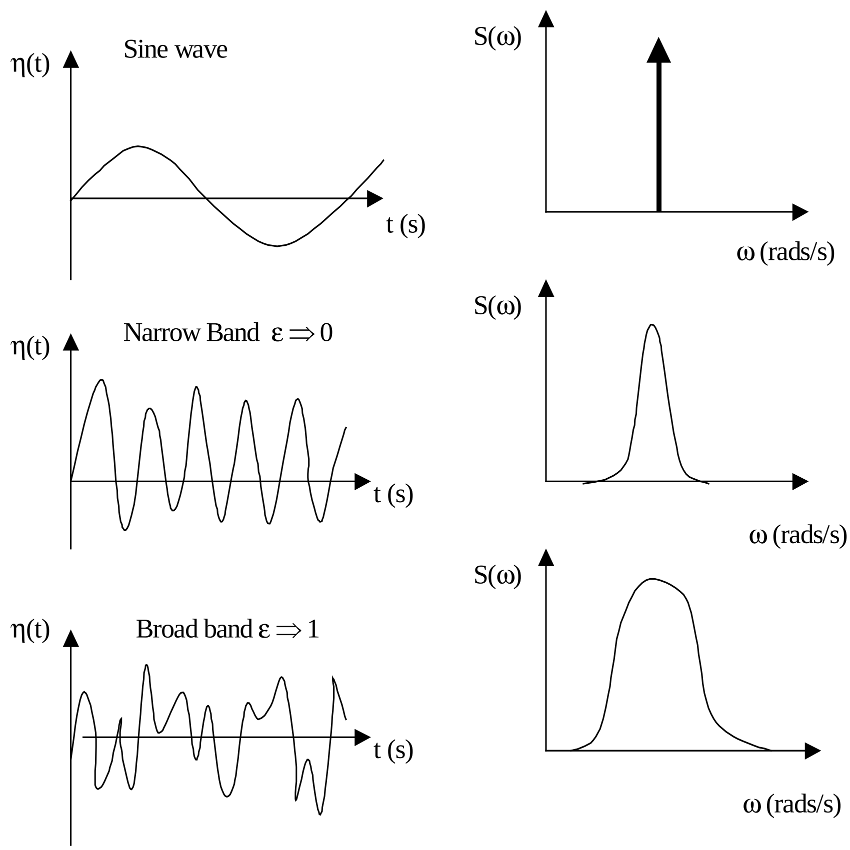

Figure 3 . Time Histories and Corresponding Spectral Shapes.

Therefore a random wave record can be represented by a plot of a^2 versus ω for all values of n, resulting in the curve illustrated in figure 3. This indicates the frequencies at which the wave energy is concentrated and those at which there is no wave energy.

Some of the most powerful techniques in mathematical analysis are various transform methods. In particular a time series x(t) can be transformed to the frequency domain, subject to certain conditions, by Fourier Integral Transforms:

$$x(\omega) = (1/2\pi)\int x(t).\ \exp(-i\omega t)\ dt \qquad 5.46$$

And its Inverse Fourier Transform:

$$x(t) = \int x(\omega).\ \exp(i\omega t)\ d\omega \qquad 5.47$$

where $i = \sqrt{-1}$.
For random waves a very useful measure of wave energy versus frequency, the **"spectral energy density function or wave spectrum",** $S_{\eta\eta}(\omega)$, can be obtained by the Fourier Transform of the Auto-correlation function $R_{\eta\eta}(\tau)$. The details are not given here but can be found in any good text on time series analysis.

$$S_{\eta\eta}(\omega) = (1/2\pi)\int R_{\eta\eta}(\tau)\ \exp(-i\omega t)\ d\tau \qquad 5.48$$

The spectral density function defined in this way is a real and even function of ω but for most engineering applications only positive frequencies are required. Therefore the spectrum is presented as a one sided, positive frequency function whose magnitudes will be twice that given by equation 39.
The inverse Fourier Transform is:

$$R_{\eta\eta}(\tau) = \int_{-\infty}^{\infty} S_{\eta\eta}(\omega)\ \exp(i\omega t)\ d\omega \qquad 5.49$$

and, importantly, $R_{\eta\eta}(\tau = 0) = \sigma_\eta^2$, the variance (standard deviation squared) of the water surface elevation fluctuations. Properly stated, <u>the spectrum is the distribution of the variance of the water surface elevation as a function of frequency.</u>
In short-crested, three-dimensional, random seas the definitions given above will result in a directional wave spectrum, $S_{\eta\eta}(\omega,\theta)$ where θ is the wave direction. From extensive analysis of wave records and by considering the basic physical processes involved, several algebraic forms for wave spectra have been developed. The most famous of these is the **Pierson-Moskowitz wave spectrum for Fully Developed Seas**, defined as:

$$S_{\eta\eta} = (\alpha g^2/\omega^5)\ \exp(-\beta\omega_o^4/\omega^4) \qquad 5.50$$

Where $\alpha = 0.0081$, $\beta = 0.74$ and $\omega_o = g/U$ where U is the characteristic wind speed (m/s).
The **JONSWOP Spectrum for Fetch Limited Seas** is similar in form to the Pierson-Moskowitz Spectrum and is given by:

$$S_{\eta\eta}(\omega) = (\alpha g^2/\omega^5)\ \exp(-\beta\omega_m^4/\omega^4)*\gamma^K \qquad 5.51$$

Where $K = \exp[-(\omega-\omega_m)^2/2\omega_m^2\sigma^2]$ 5.52
$\beta = 1.25$ $\sigma = 0.07$ for $\omega \le \omega_m$ $\sigma = 0.09$ for $\omega \ge \omega_m$ 5.53
γ is the "peak enhancement factor"
ω_m is the frequency of the spectral peak.

and ω_m and α are functions of the wind speed and fetch.

The "moments", $m'_{j,}$ of a spectrum define its general shape, where

$$m'_j = \int_0^\infty \omega^j S_{\eta\eta}(\omega)\, d\omega \qquad 5.54$$

The "Spectral Bandwidth", $\varepsilon^2 = 1 - ((m'_2)^2/(m'_o\, m'_4))$ 5.55

This parameter gives rise to the terms "narrow-band" process and "wide band" process, the former implying a signal with a narrow range of frequencies, the latter one with a wide range of frequencies, as illustrated in figure 4.
Care must be taken in calculating the moments of a spectrum because the ω^j term amplifies the high frequency "tail" of the spectrum and can lead to instabilities in the numerical values of the moments. In practice a random wave record with a bandwidth as high as 0.8 will appear to be relatively uniform.
A number of software data analysis packages contain programmes which take the wave record as input and use a "Fast Fourier Transform" routine to calculate the spectrum.

5.3. WAVES IN SHALLOW WATER.

5.3.1. Introduction.

If design wave conditions have been derived from data in "deep" water (water depth, d, greater than half the wave length, L) the conditions at a shallow water site will be influenced strongly by shallow water wave process. If the wave data was recorded in shallow water near to, but not at the site they will have to be transformed to the equivalent "deep" water conditions and then transformed again to the site location, ("out and in again").

There are several physical processes that take place as waves travel into shallow water: shoaling, refraction, bed friction and percolation, diffraction at obstacles and, eventually, wave breaking at the outer limits of the surf zone.

This section of the notes deals with those processes.

5.3.2 Wave Shoaling.

Linear Wave theory gives the phase velocity , C, of a wave as:

$$C = [(gL/2\pi)\tanh(2\pi d/L)]^{1/2} \qquad 5.56$$

where L is the wave length in a water depth, d.
In "deep" water $2\pi d/L \Rightarrow \infty$ and $\tanh(2\pi d/L) \Rightarrow 1.0$ so the phase velocity becomes:

$$C = (gL_o/2\pi)^{1/2} \qquad \text{subscript o denoting "deep" water.} \qquad 5.57$$

The energy in a wave travels at the "group velocity", $C_{g,}$, which is related to the phase velocity, C. In shoaling water the group velocity changes at a different rate to the phase velocity and there is a corresponding change in wave height.
A **Shoaling Coefficient, K_d,** can be calculated so that

$$H = K_d H_o \qquad 5.58$$

From linear wave theory:

$$K_d = [\tanh(2\pi d/L)\,(1 + \{(4\pi d/L)/(\sinh(4\pi d/L)\}]^{1/2} \qquad 5.59$$

K_d is a function of water depth and not of wave height. In deep water $K_d = 1.0$, it is a minimum of 0.91 at $d/L_o \approx 0.15$ and as the water depth reduces it increases without limit. In reality the wave height is limited by wave breaking.

5.3.3. Wave Refraction.

The phase speed of the wave reduces in shallow water according to equation 1. If a wave approaches a coastline at an angle, part of the wave moves into shallow water earlier than the remainder, that part of the wave decelerates and the wave changes direction. This process is termed wave refraction and it

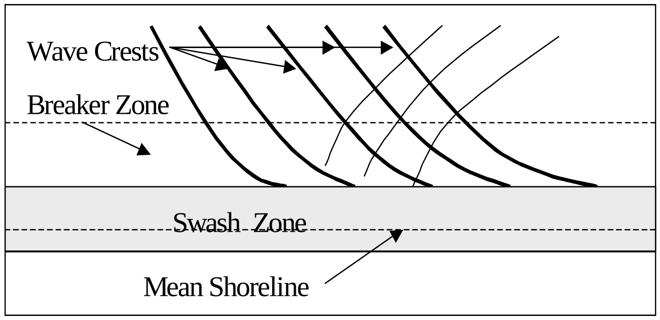

Figure 1. Wave Refraction with Parallel Bed Contours

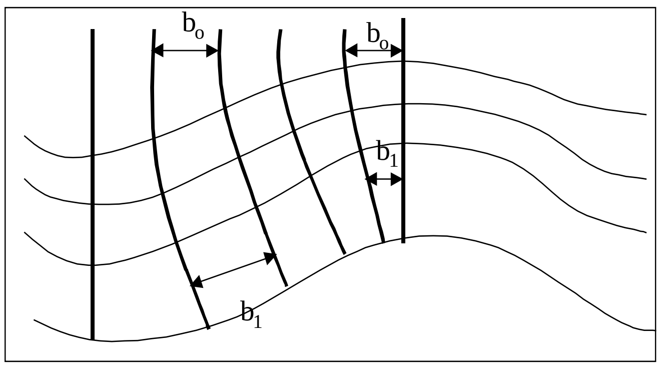

Figure 2. Wave Refraction over Curved Contours

results in changes in both wave direction and wave height. Figure 1 illustrates the refraction of waves over straight and parallel sea bed contours. Figure 2 illustrates wave orthogonals – lines at right angles to the wave crests – on a curved coastline.

Defining α as the angle between the wave crest and the contour line, Snell's Law gives:

$$C_1 / C_2 = \sin \alpha_1 / \sin \alpha_2 \qquad 5.60$$

The energy flux per unit length of wave crest is proportional to $C_g H^2$ so over a length b,
$C_g H^2 b$ is constant, where b is the distance between two orthogonals. Therefore:

$$H_1/H_2 = (C_{g1}/C_{g2})^{1/2} . (b_1/b_2)^{1/2} \qquad 5.61$$

The group velocity ratio is the shoaling coefficient, K_D, and $(b_1/b_2)^{1/2}$ is the **Refraction Coefficient, K_R**. Because sea-bed levels are usually irregular it is normally necessary to solve for wave refraction by using a numerical model but it can be done by manual plotting. The former is now routine, with the output being given in the form:

$$H = K_R\, H_o \qquad 5.62.$$

In the case of a straight shore with parallel contours $b_1/\cos\alpha_1 = b_2/\cos\alpha_2$. In the same way as the shoaling coefficient, K_R does not depend on wave height. Thus calculations can be done for unit deep water wave height and the K_R applied to other heights.

Combining shoaling and refraction:

$$H = K_d\, K_R\, H_o \qquad 5.63$$

5.3.4. Friction and Percolation.

Wave energy loss due to friction and percolation at the sea bed will obviously result in a reduction in wave height. So one can write:

$$H = K_F\, H_o \qquad 5.64$$

in terms of a coefficient to account for the change in wave height due to friction.. However, the shear stress acting on the sea bed will be proportional to the square of the local, oscillatory flow velocity. The flow-induced shear stress times the local velocity equals the work done which has to be integrated over a wave cycle. Therefore K_F will itself be a function of wave height, wave period, water depth and a friction factor analogous to a Chezy coefficient. This extends the computation time because the numerical model must be run for all wave periods and all wave heights in the wave climate. Neglecting the effects of friction will lead to an overestimate (safe) of the shallow water wave heights.
A similar coefficient can be derived for percolation – the flow of water in and out of the bed under wave action. This is a complex process and depends critically on the nature of the sea bed material. There is still limited understanding of this process but some guidance is available in the literature.

5.3.5. Wave Diffraction.

Wave diffraction is the scattering of an incident wave field by an obstacle. The most comment examples are breakwaters and large diameter offshore structures (for the latter see notes on wave loading). Figure 4 illustrates the way in which waves diffract around the end of a semi-infinite breakwater in constant water depth. Formal solutions for the wave diffraction coefficients are available in terms of Fresnel integrals – a parallelism with the diffraction of light waves. The wave height incident to the breakwater is denoted by H_I, then the wave height at any location influenced by the scattered waves is given by:

$$H(x,y) \text{ or } H(r,\theta) = D_d\, H_I \qquad 5.65$$

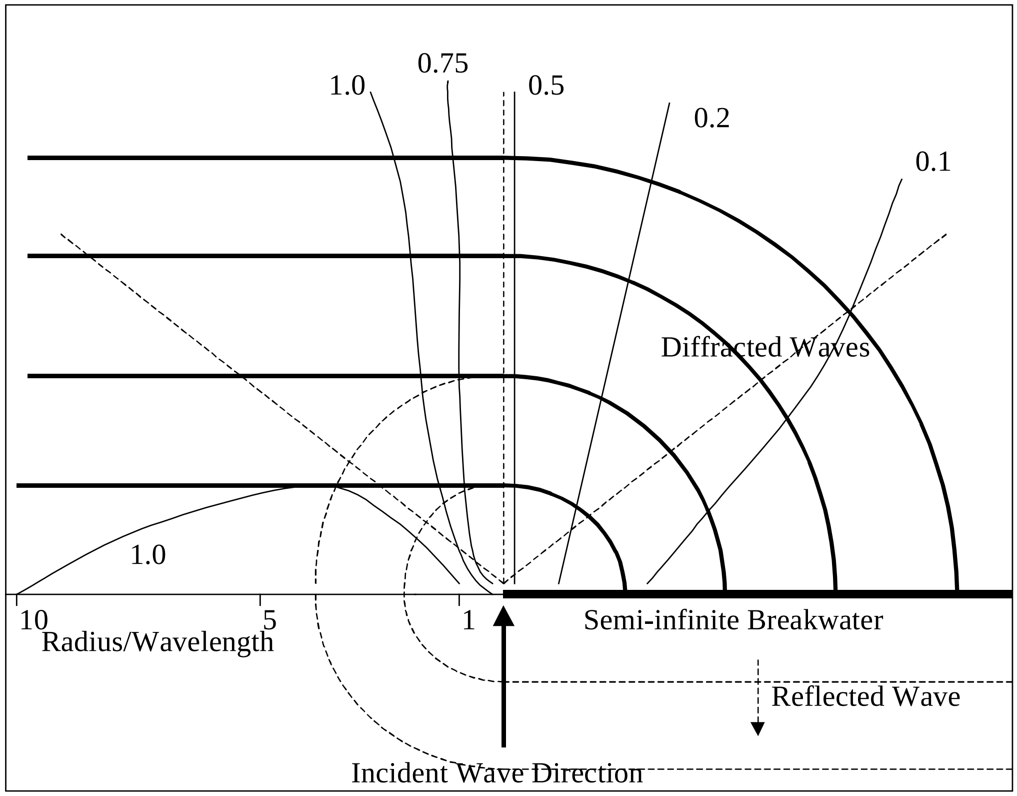

Figure 3. Wave Diffraction with Indicative Diffraction Coefficients

D_d, the diffraction coefficient, is a function of (x,y) or (r,θ) in Cartesian or Polar coordinates, respectively, and is independent of wave height. Solutions for D_d are available in the literature, e.g., SPM, in the form of polar plots of the coefficient for a range of incident wave angles relative to the line of the breakwater. These plots also include solutions for diffraction through a gap between two breakwaters, non-dimensionalised in terms of b/L where b is the width of the gap and L is the local wave length.

5.3.6. Nearshore Wave Climate.

The combined influence of these shallow water processes can be written as:

$$H = K_R\, K_d\, K_F\, K_P\, D_d\, H_o \qquad 5.66$$

If wave reflection occurs this can be taken into account by applying a reflection coefficient which will depend on the properties of the reflecting boundary. The direction of the reflected waves is given by “the angle of incidence equals the angle of reflection”.
Note that reflections will produce a complex wave pattern in the form of “standing waves”.

Knowing the offshore wave climate, in terms of H_S, T_Z and θ_o, the products of the coefficients can be applied to each component of the wave climate to give the wave conditions at a shallow water location – but note that wave directions may change significantly.

As mentioned previously, if wave measurements were available in a location in shallow water the inverse of the above transformation would have to be applied to give deep water wave conditions which could then the re-transformed to the specific site for design.

5.3.7. Summary.

Given a deep water wave climate, procedures exists to transfer that climate to a shallow water location – just outside the breaker line. The wave climate is normally given in terms of H_S, T_Z and θ_o but it is equally possible to transform a direction wave spectrum from deep to shallow water. The latter procedure would be needed if a dynamically responsive system such as a floating platform was to be located in shallow water and it was necessary to determine its response to wave action.

Eventually the waves break and if they do so at an angle to the sea bed contours they create a current travelling in an alongshore direction. It is these currents which can be turned to run offshore that create "rip currents" which are dangerous to bathers.

2.3.8. Wave Breaking.

Clearly waves eventually break on beaches and sloping structures, travelling as "bores", similar in nature to a moving hydraulic jump. As a general indicator a wave will break in shoaling water when its height is approximately 0.78 times the water depth.

$$H_b = 0.78\ h_b \qquad 5.67$$

On steep beaches this becomes $H_b = 1.0h_b$ and in random waves some recent work gives:

$$H_{sb}/h = 0.707 + 0.57 \tanh(23H_{sb}/L_o) \qquad 5.68$$

where Hsb is the significant wave height at breaking.Equaiton 14 has to be solved iterativley.

5.3.9. Wave-induced Longshore Currents.

In the full momentum equations for wave motion in shallow water there are six mechanisms involved:

1. Radiation Stresses
2. Pressure gradients due to mean water level variations – the latter termed wave set-up and set-down.
3. Mean bottom friction due to currents and waves.
4. Lateral mixing due to turbulence and the horizontal shear of currents.

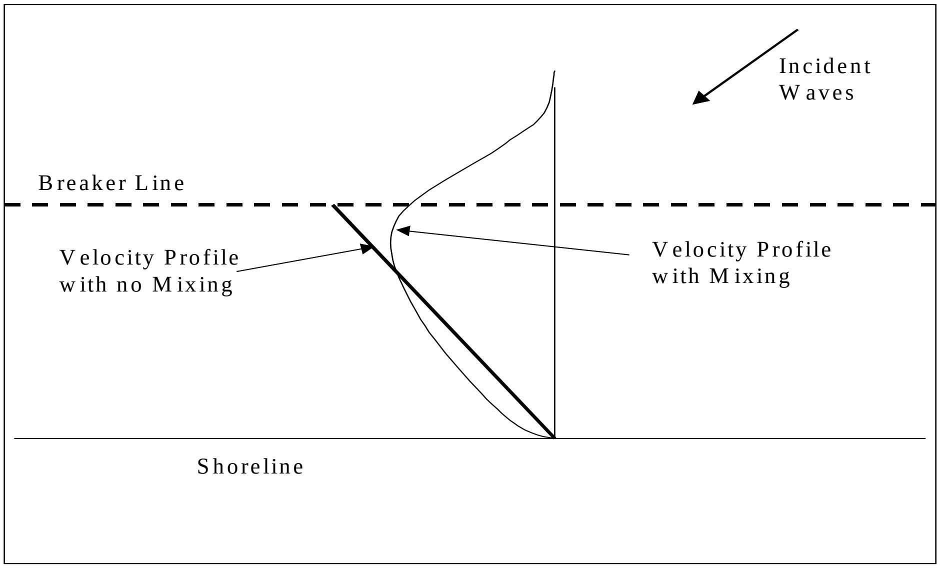

Figure 3. Longshore Wave-induced Current

5. The inertia of the water column – because it experiences accelerations.
6. Interactions between the waves and the currents.

In these notes we consider only a part of the physical processes involves, namely the consequence of the balance between radiation stresses and bed friction, because this results in wave-generated longshore currents and these influence the behaviour of beaches. The Radiation Stress represents the time averaged force that the wave exert on the water column. Longuet-Higgins, in 1970, was the first to solve this problem and derived an equation for the wave-induced current, based on linear wave theory inside the surf zone. Despite much subsequent research there are still unsolved aspects of these currents – not surprisingly considering the complexity of the flow in the surf zone. Waves arriving at the coastline at any angle other than zero will generate these currents but any variations in the height of breaking waves in the longshore direction will also generate currents.
Ignoring the mixing processes due to turbulent flow and the shear stresses that this generates, an estimate of the lonshore wave-induced velocity at the breaker line is given by:

$$V = (5\pi/16\, f_c)\ \gamma(gh_b)^{1/2}\ s.\sin\theta_b \qquad 5.69$$

where f_c is a friction factor, γ is the wave breaking index and s is the beach slope. The longshore velocity increases linearly from zero at the shoreline to V at the breaker line.
Measurements have shown that the longshore current has a maximum somewhat shorewards of the outer limit of breaking, (note that in random waves the breaker point will vary) as illustrated in figure 4 below.. This is due to turbulent mixing and the fact that wave heights in a real sea are not uniform. The longshore current is, of course, capable of transporting sediment – see notes on sediment transport.
